ANALOG DEVICES

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AN-281
APPLICATION NOTE

Passive and Active Analog Filtering

Filtering is an important part of analog signal processing. Filtering can be used to reduce unwanted signals, limit bandwidth, help recover wanted signals, minimize aliasing in sampled data systems, and smooth the output of DACs. There are five classes of filters. *Lowpass* filters pass all frequencies below the cutoff frequency and block all frequencies above the cutoff frequency. *Highpass* filters are the inverse of the low-pass filters. They block the low frequencies and pass those above the cutoff frequency. *Bandpass* filters pass those frequencies between the lower cutoff and upper cutoff frequencies and reject all others. *Bandstop* filters are the inverse of bandpass filters. They reject frequencies between the cutoff frequencies and pass all others. *Allpass* filters pass all frequencies equally but introduce a predictable phase delay to the signal.

CLASSES OF PASSIVE AND FILTERS

- **Lowpass**
- **Highpass**
- **Bandpass**
- **Bandstop**
- **Allpass**

Figure 2.56

Traditional filters were passive, that is designed with no active elements. Active components were too costly and had very poor performance characteristics. Inductors, capacitors, and resisters were used to synthesize the filter. This approach has several difficulties because inductors become physically large for low frequency filters and have poor characteristics at high frequencies. There is a great deal of interaction between the different sections of the filter. Impedance levels must be precisely controlled. Close component tolerances are difficult to manufacture and maintain. Despite these limitations passive filters are still dominant at high frequencies, primarily due to dynamic performance limitations of op amps.

PASSIVE FILTERS

- **Designed with Inductors, Capacitors, Resistors**
- **Large Inductors Required for Low Frequency Filters**
- **Interaction Between Filter Stages**
- **Component Tolerances Difficult to Manufacture and Maintain**
- **Still the Only Solution at High Frequencies Due to Active Component Limitations**

Figure 2.57

Active filters answer some of the limitations of the passive filter by offering isolation between stages and eliminating the need for inductors. Their use at high frequencies is limited by the dynamic performance of the active elements.

ACTIVE FILTERS

- **Eliminate Need for Inductors**
- **Good Interstage Isolation**
- **High Frequency Use Limited by Op Amp Dynamic Performance**

Figure 2.58

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A filter can be specified in terms of five parameters as shown in Figure 2.59. The *cutoff frequency* F_c is the frequency at which the filter response leaves the error band (or the -3dB point for a Butterworth filter). The *stopband frequency* F_s is the frequency at which the minimum attenuation in the stopband is reached. The *passband ripple* A_{max} is the variation (error band) in the passband response. The *minimum passband attenuation* A_{min} defines the signal attenuation within the stopband. The *order* M of the filter is the number of poles in the transfer function.

KEY FILTER DESIGN PARAMETERS

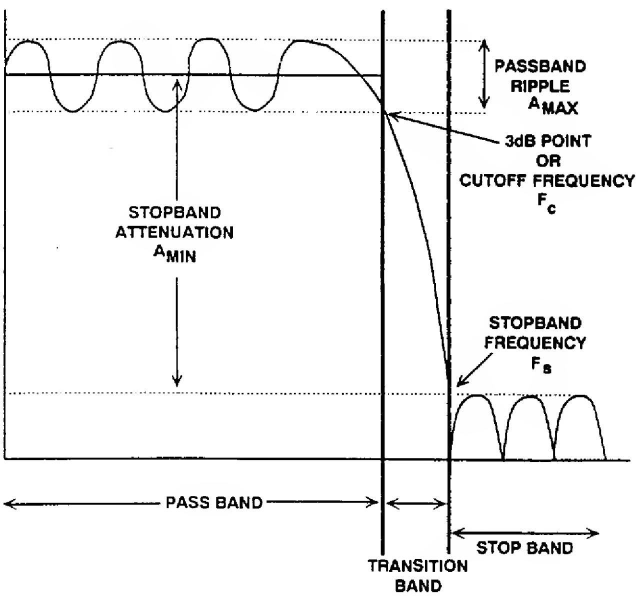


Figure 2.59

FILTER SPECIFICATIONS

- Cutoff Frequency, F_C
- Stopband Frequency, F_S
- Passband Ripple, A_{MAX}
- Stopband Attenuation, A_{MIN}
- Filter Order, M

Figure 2.60

Typically, one or more of the above parameters will be variable. For instance, if you were to design an antialiasing filter for an ADC you will know the cutoff frequency, the stopband frequency, and the minimum attenuation. You can then go to a chart or computer program to determine the other parameters.

There are many transfer functions that may satisfy the requirements of a particular filter. The *Butterworth* filter is the best compromise between attenuation and phase response. It has no ripples in the passband or the stopband and is called the *maximally flat filter* because of this. The Butterworth filter achieves its flatness at the expense of a relatively wide transition region from passband to stopband.

The *Chebyshev* filter has a smaller transition region than the same-order Butterworth filter, but it has ripples in either its passband or stopband. This filter gets its name because the Chebyshev filter minimizes the height of the maximum ripple—this is the Chebyshev criterion.

The Butterworth filter and the Chebyshev filter are all-pole designs. By this we mean that the zeros of the transfer function are at one of the two extremes of the frequency range (0 or ∞). For a lowpass filter the zeros are at f = ∞. We can add finite frequency transfer function zeros as well as poles to get an *Elliptical Filter*. This filter has a shorter transition region than the Chebyshev filter because it allows ripple in both the stopband and passband. The Elliptical filter also has degraded phase (time domain) response.

These are by no means all possible transfer functions, but they do represent the most common.

POPULAR FILTER DESIGNS

- **Butterworth: All Pole, No Ripples in Passband or Stopband, Maximally Flat Response**
- **Chebyshev: All Pole, Ripple in Passband, Shorter Transition Region than Butterworth for Given Number of Poles**
- **Elliptical: Ripple in Both Passband and Stopband, Shorter Transition Region than Chebyshev, Degraded Phase Response, Poles and Zeros**

Figure 2.61

Once the order of the filter and the specifications of filter have been determined, the design charts (see Reference 10) or computer programs are consulted, and the linear and quadratic factors of poles for the transfer function are determined. All filters, regardless of order, are made up of one- or two-pole sections. The single pole section is defined by its resonant frequency, which is the -3dB point. The pole pair in a two-pole filter section is defined by its resonant frequency (F_o) and Q, which indicates the peaking of the section. Sometimes alpha (α) is used instead of Q (Q=1/α).

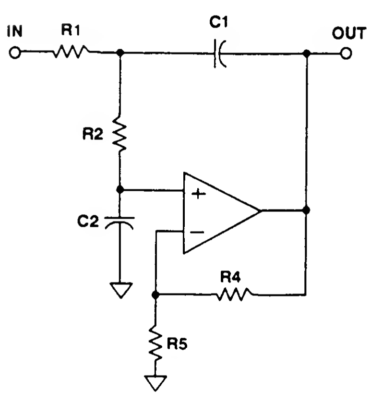
Armed with the various values F_o and Q, you then choose the configuration for the realization of the filter: Butterworth, Chebyshev, or Elliptical.

For passive filters, these values, along with the filter characteristic impedance determine the inductor, capacitor, and resistor values.

For active filters, you must decide which of the realizations you are going to use. The three most common are the *Sallen-Key* (voltage controlled voltage source), *multiple feedback*, and *state variable*. Each realization has its own advantages and disadvantages.

The Sallen-Key configuration shown in Figure 2.62 is the least dependent on the performance of the op amp, and the signal phase is maintained. For this filter the ratio of the largest resistor value to the smallest resistor value and the ratio of the largest capacitor value to the smallest capacitor value is low. The frequency term and Q terms are somewhat independent, but they are very sensitive to the gain parameter. The Sallen-Key is very Q-sensitive to element values for high Q sections. The design equations are also given in Figure 2.62.

VOLTAGE CONTROLLED VOLTAGE SOURCE (SALLEN-KEY) REALIZATION



H = Circuit Gain Below Cutoff

α = Damping Ratio = 1/Q

F_O = Cutoff Frequency

Choose C1

$K = 2\pi F_O C1$

$M = \frac{\alpha^2}{4} + H - 1$

$C2 = M\,C1$

$R1 = \frac{2}{K\alpha}$

$R2 = \frac{\alpha}{2MK}$

Choose R5

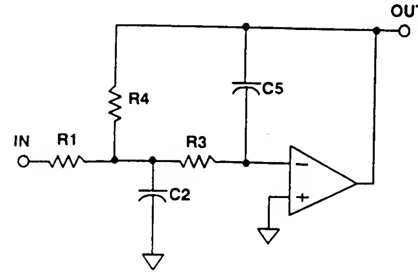
$R4 = R5(H - 1)$

For H = 1, R4 = 0, R5 = Open

Figure 2.62

The multiple feedback realization shown in Figure 2.63 uses an op amp in the inverting configuration. The dependence on the op amp parameters are greater than in the Sallen-Key realization. It is hard to generate high Q sections due to the limitations of the open loop gain of the op amp. The maximum to minimum component value ratios are higher than in the Sallen-Key realization. The design equations are also given in Figure 2.63.

MULTIPLE FEEDBACK REALIZATION



F_O = Cutoff Frequency

α = Damping Ratio = 1/Q

H = Absolute Value of Circuit Gain

Choose C5

$K = 2\pi F_O C1$

$C2 = \frac{4C5}{\alpha^2}(H + 1)$

$R1 = \frac{\alpha}{2HK}$

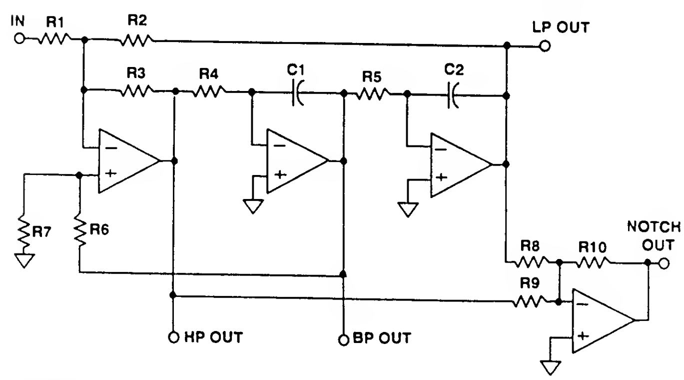
$R3 = \frac{\alpha}{2K(H + 1)}$

$R4 = HR1$

Figure 2.63

The state-variable realization shown in Figure 2.64 offers the most precise implementation, at the expense of many more circuit elements. All parameters can be adjusted independently, and lowpass, highpass, and bandpass outputs are all available simultaneously. The gain of the filter is also independently variable. Since all parameters of the state variable filter can be adjusted independently, component spread is minimized. Also variations due to temperature and component tolerances are minimized. The design equations for the state variable filter are given in Figure 2.64.

STATE VARIABLE REALIZATION



LOWPASS GAIN = -R2/R1

HIGHPASS GAIN = -R3/R1

$$\text{BANDPASS GAIN} = \frac{R6 + R7}{R1R7\left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}\right)}$$

$$F_O = \frac{1}{2\pi}\sqrt{\frac{R3}{R2 \cdot R4 \cdot R5 \cdot C1 \cdot C2}}$$

$$Q = \frac{1}{\alpha} = \frac{R6 + R7}{R7}\left(\frac{1}{\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}}\right)\sqrt{\frac{R4 \cdot C1}{R2 \cdot R3 \cdot R5 \cdot C2}}$$

FOR NOTCH FREQUENCY = F_Z

FOR $F_O = F_Z$, $\frac{R2 \cdot R9}{R3 \cdot R8} = 1$

FOR $F_O > F_Z$, $\frac{R2 \cdot R9}{R3 \cdot R8} < 1$

FOR $F_O < F_Z$, $\frac{R2 \cdot R9}{R3 \cdot R8} > 1$

$$\frac{F_Z^2}{F_O^2} = \frac{R2 \cdot R9}{R3 \cdot R8}$$

Figure 2.64

Another active filter realization that has recently become more popular is the *Frequency Dependent Negative Resistor* (FDNR), which is a subset of the *General Impedance Converter* (GIC). In the FDNR the passive realization goes through a transformation by

1/s. Therefore inductors, whose impedance is sL, transform into a resistor of value L. Similarly, a resistor of value R becomes a capacitor of value R/s. A capacitor of impedance 1/sC transforms into a frequency dependent variable resistor, which is given the designation D. Its impedance is $1/s^2C$. The transformations to the FDNR configuration and the GIC implementation of the D element are given in Figure 2.65.

FREQUENCY DEPENDENT NEGATIVE RESISTOR 1/S IMPEDANCE TRANSFORMATION

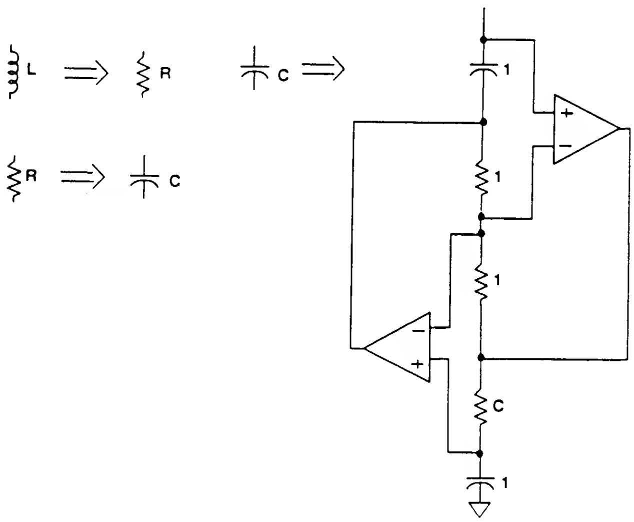


Figure 2.65

The advantage of the FDNR realization is that there are no op amps in the signal path which can add noise. This realization is also relatively insensitive to component variation. The advantages of the FDNR come at the expense of an increase in the number of components required.

For all of the realizations discussed above, the tabulated filter values are in terms of the lowpass function normalized to a frequency of 1 radian/second with an impedance level of 1. To realize the final design, the filter values are scaled by the appropriate frequency and impedance.

Similarly, the lowpass prototype is converted to a highpass filter by scaling by 1/s in the transfer function. In practice this amounts to capacitors becoming inductors with a value 1/C and inductors becoming capacitors with a value of 1/L for passive designs. For active designs resistors become capacitors with a value of 1/R, and capacitors become resistors with a value of 1/C.

Transformation to the bandpass response is a little more complicated. If the corner frequencies of the bandpass are widely separated (by more than 2 octaves) the filter is made up of separate lowpass and highpass sections. In the case of a narrowband bandpass filter the design is much more complicated and is usually done using a computer program or design tables.

SOME ACTIVE FILTER REALIZATIONS

- **Sallen-Key: Good Phase Response, Least Dependent on Op Amp Performance, Sensitive to Element Values for High Q Sections**
- **Multiple Feedback: Less Sensitive to Element Values, High Q Sections Difficult due to Op Amp Open Loop Gain Limitations**
- **State-Variable: Most Precise, More Components, All Parameters Independently Adjustable**
- **Frequency Dependent Negative Resistance (FDNR): Op Amps not in Signal Path, More Components, Relatively Insensitive to Component Variations**

Figure 2.66

Antialiasing Filter Design Example

We will now design a passive and active antialiasing filter based upon the same specifications. The active filter will be designed in four realizations: Sallen-Key, multiple feedback, state variable, and Frequency Dependent Negative Resistance (FDNR). We choose the Butterworth filter in order to give the best compromise between attenuation and phase response.

The specifications for the filter are as follows:

ANTIALIASING FILTER SPECIFICATIONS

- **Cutoff Frequency F_C = 8kHz**
- **Stopband Attenuation F_S at 50kHz = 70dB**
- **Best Balance Between Attenuation and Phase Response**
- **Choose Butterworth Design**
- **From Design Charts, for f = 6.25 (50kHz/8kHz), M = 5**

Figure 2.67

Consulting the design charts (Reference 10, p. 82), we see that for 70dB of attenuation at a frequency of 6.25 (50kHz/8kHz) a

fifth order filter is required.

We now consult the tuning tables (Reference 10, p. 341) and find:

ALPHA AND F_0 VALUES FROM TUNING TABLES

STAGE	ALPHA	F_0
1	- - -	1.000
2	1.618	1.000
3	0.618	1.000

Figure 2.68

The first stage is a real pole, thus the lack of an alpha value. It should be noted that this is not necessarily the order of implementation in hardware. In general you would typically put the real pole last and put the second order sections in order of decreasing alpha (increasing Q).

For the passive design we will choose the zero input impedance configuration. From the design table (Reference 10, p. 313) we find the following normalized values for the filter:

NORMALIZED PASSIVE FILTER VALUES FROM TABLES

L1 = 1.5451	C2 = 1.6944
L3 = 1.3820	C4 = 0.8944
L5 = 0.3090	

Figure 2.69

These values are for a 1 rad/second filter with a 1 ohm termination. To scale the filter we divide all reactive elements by the desired cutoff frequency, 8kHz (50265 rad/sec). We also need to scale the impedance. For this example, we choose a value of 1000 ohms. To scale the impedance we multiply all resistor and inductor values and divide all capacitor values by the impedance scaling factor. After scaling, the circuit looks like Figure 2.70.

EXAMPLE FILTER PASSIVE IMPLEMENTATION

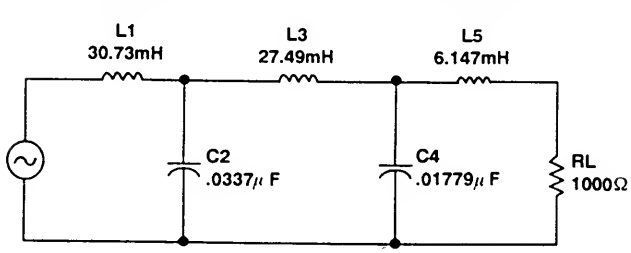


Figure 2.70

For the Sallen-Key active realization, we use the design table shown in Figure 2.62. The values for C1 in each section are chosen to give reasonable resistor values. The implementation is shown in Figure 2.71. For the Sallen-Key realization to work correctly, it is assumed to have a zero-impedance driver and a return path for dc. Both of these criteria are approximately met when you use an op amp to drive the filter.

EXAMPLE FILTER SALLEN-KEY IMPLEMENTATION

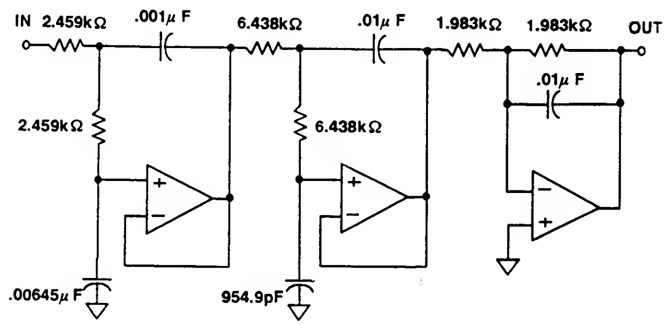


Figure 2.71

Figure 2.72 shows a multiple feedback realization of our filter. It was designed using the equations in Figure 2.63.

EXAMPLE FILTER MULTIPLE FEEDBACK IMPLEMENTATION

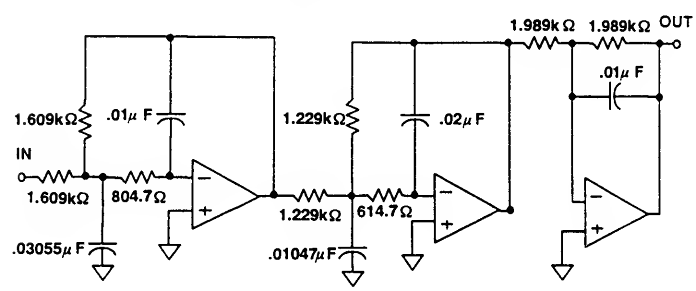


Figure 2.72

The state variable realization is shown in Figure 2.73, and the Frequency Dependent Negative Resistance (FDNR) realization is shown in Figure 2.74. In the conversion process from passive to FDNR, the D element is normalized for a capacitance of 1F. We then scale the filter to a more reasonable value (0.01µF in this case).

In all of the filters above the values shown are the exact calculated values. These exact values are rarely obtainable. We must therefore either substitute the nearest standard value or use series/parallel combinations. Any variation from the ideal values will cause a shift in the filter response char-

EXAMPLE FILTER
STATE VARIABLE IMPLEMENTATION

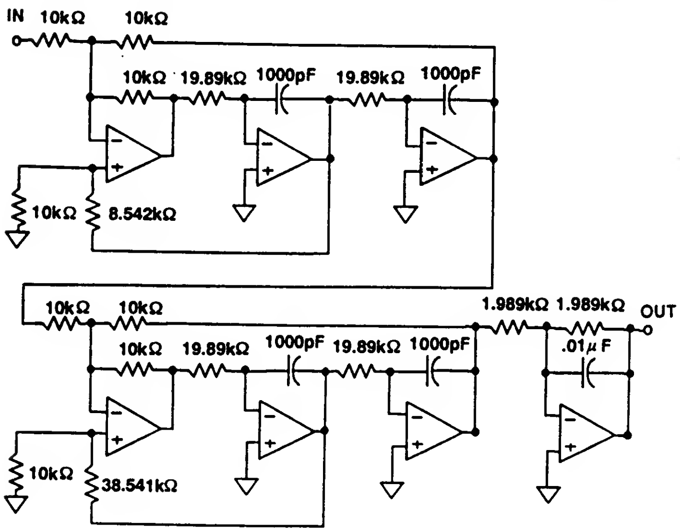


Figure 2.73

EXAMPLE FILTER
FDNR IMPLEMENTATION

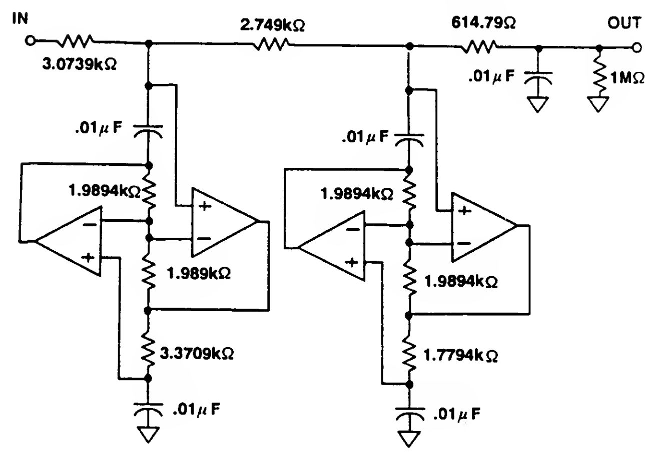


Figure 2.74

acteristic, but often the effects are minimal. The computer can be used to evaluate these variations on the overall performance and determine if they are acceptable.

In active filter applications using op amps, the dc accuracy of the amplifier is often critical to optimal filter performance. The amplifier's offset voltage will be passed by the filter and may be amplified to produce excessive output offset. For low frequency applications requiring large value resistors, bias currents flowing through these resistors will also generate an output offset voltage.

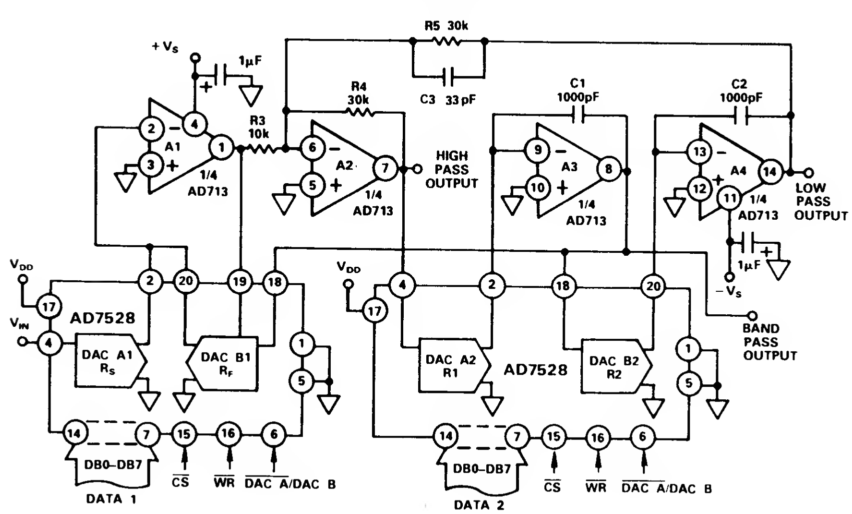
In addition, at higher frequencies, an op amp's dynamics must be carefully considered. Here, slewrate, bandwidth, and open loop gain play a major role in op amp selection. The slewrate must be fast as well as symmetrical to minimize distortion.

A Programmable State Variable Filter

A realization of a programmable state variable filter using DACs is shown in Figure 2.75. DACs A1 and B1 control the gain and Q of the filter characteristic, while DACs A2 and B2 must accurately track for the simple expression for f_c to be true. This is readily accomplished using two AD7528 DACs and one AD713 quad op amp. Capacitor C3 compensates for the effects of op amp and gain-bandwidth limitations.

This filter provides lowpass, highpass, and bandpass outputs and is ideally suited for applications where digital control of filter parameters is required. The programmable range for component values shown is f_c = 0 to 15kHz, and Q = 0.3 to 4.5.

A PROGRAMMABLE STATE
VARIABLE FILTER CIRCUIT



CIRCUIT EQUATIONS

$C_1 = C_2 , R_1 = R_2 , R_4 = R_5$

$$f_C = \frac{1}{2\pi R_1 C_1}$$

$$Q = \frac{R_3}{R_4} \cdot \frac{R_F}{R_{FBB1}}$$

$$A_0 = -\frac{R_F}{R_S}$$

NOTE:
DAC equivalent resistance equals

$$\frac{256 \times (\text{DAC Ladder resistance})}{\text{DAC Digital Code}}$$

Figure 2.75

Figure 2.76 shows a 7-pole antialiasing filter for a 2x oversampling (88.2kSPS) digital audio application. This filter has less than 0.05dB passband ripple and 19.8 ± 0.3μs delay, dc-20kHz. The filter will handle a 5V rms signal (V_s = ±15V) with no overload at any internal nodes. The frequency response of the filter is shown in Figure 2.77.

20kHz FDNR AUDIO ANTIALIASING FILTER

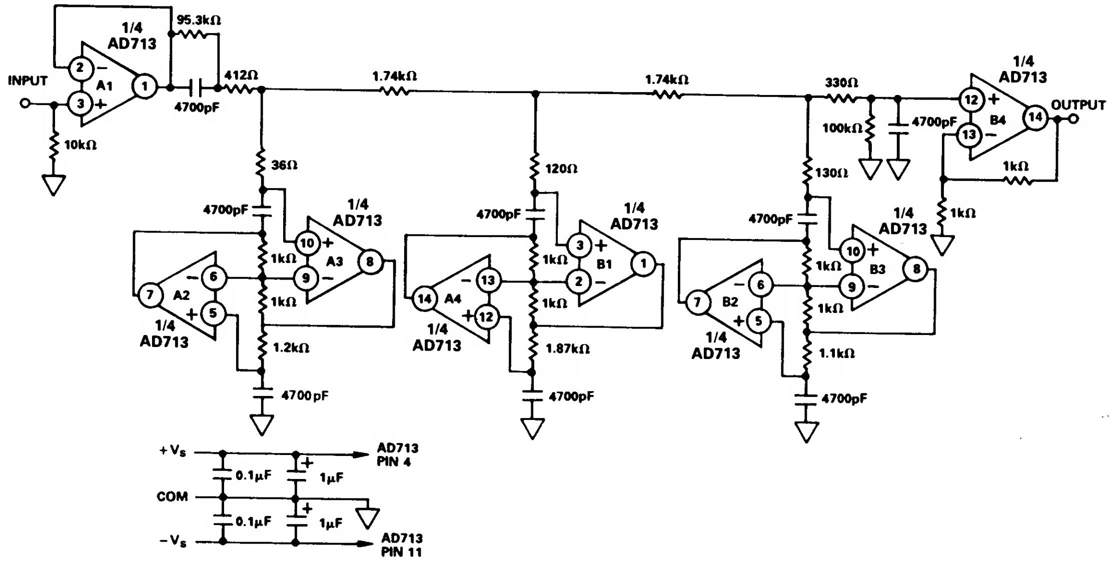


Figure 2.76

AUDIO ANTIALIASING FILTER RESPONSE

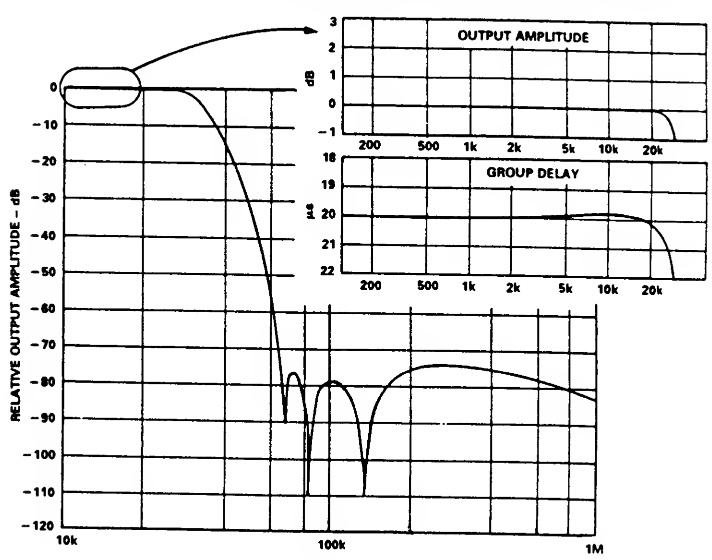


Figure 2.77

A Wideband Sallen-Key Filter

Figure 2.78 shows an AD843 FET input op amp used in a 1MHz Sallen-Key filter. This circuit also works well with the AD841, AD845, or AD847. The circuit is designed to be a maximum-flatness filter with a Q of 0.575 and a dc gain of 1.26. The frequency response of the filter to a 0dBm input signal is shown in Figure 2.79.

1 MHz SALLEN KEY FILTER

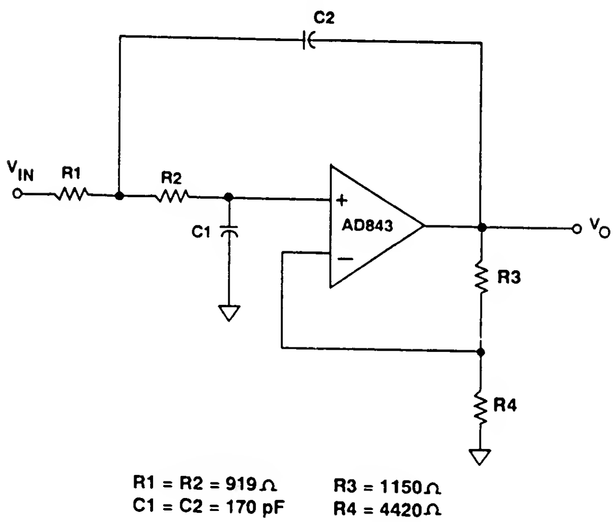


Figure 2.78

SALLEN-KEY SMALL SIGNAL FREQUENCY RESPONSE

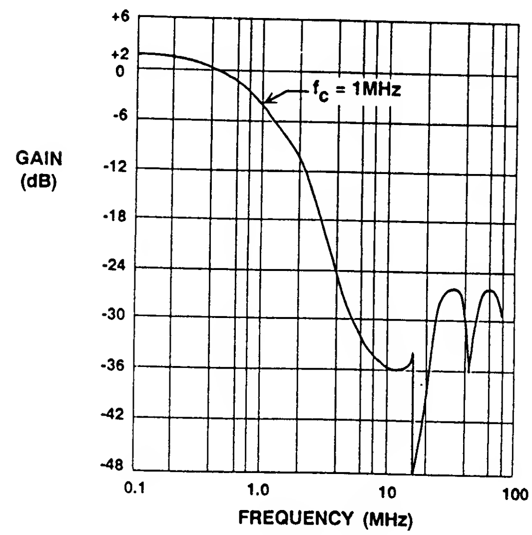


Figure 2.79